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APPLICATION OF THE BOUNDARY ELEMENT METHOD TO THE SIMULATION IN TUNNELING

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Abstract

The paper presents special solution techniques that need to be applied with the Boundary Element Method (BEM) for the realistic simulation of tunnel excavation. The work is based on the original ideas published by Venturini in 1983. It will be explained how the BEM is adapted to deal efficiently with heterogeneous and non-linear ground behavior, the modeling of ground support and sequential excavation. Examples of application are presented.

Keywords: Simulation. Boundary Element Method. Tunneling.

1 INTRODUCTION

In 1983 the book “Boundary Element Methods in Geomechanics” authored by Sergio Venturini was published in the Lecture Notes in Engineering series. At this time it was the most advanced book on the topic and it inspired many, including myself. In his work one could see the beauty of the BEM when applied to problems in geomechanics. Many of the original ideas presented in that book served as starting point for the methods presented here, some of which are tangential to the research carried out by the very active group under Sergio’s leadership in São Carlos.

It was discovered very early on that, because in the BEM unknowns only exist on the boundary the presence of forces in the continuum (“body forces”) would require further development of the method. Under the term “body forces” various effects may be summarized: Self weight, centrifugal forces, swelling and non-linear behavior of the material. In addition, algorithms may be developed based on “body forces” that allow an efficient treatment of “inclusions” in the continuum. These inclusions may be zones of different material, ground improvement techniques or supports.

The problem with body forces is that, if they are present, in addition to the boundary integrals, a volume integral appears. At the same time as Venturini published his work on geomechanics, Telles published a method for dealing with inelastic behavior. Both were students of Carlos Brebbia, who was pushing the BEM at that time. The initial stress method, that was introduced for the Finite Element method, some time previously, was used. The initial stresses acted as body forces and it was suggested to use cells for the evaluation of these integrals. This was then applied to tunneling problems by Venturini using no tension, multi-laminate and time dependent material laws. Since that time, the development of the BEM in geomechanics was slow, mainly because of the limited number of groups working on the method as compared with the FEM. Of course Sergio continued his excellent work in São Carlos and because of our mutual interest we established some cooperation between our groups at a fairly early stage. There were many events that we participated and I can remember very well our reunions in Mar de Plata, Margarita island and one very memorable “Heurigen” near Vienna. He was always an inspiration to listen too and had a very special sense of humor. When I returned to Austria from Australia in 1993 I was already very much working in the BEM field but always coupling the BEM with the FEM wherever body forces appeared. The program that I developed at that time, BEFE was

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applied to many interesting problems world wide, but I very soon discovered that there must be a better way of dealing with these problems than coupling with FEM and using multiple regions for inclusions. On this note the project of the development of BEFE++ was started. The idea was to develop a completely new program with teamwork rather than one single person, as was the case with the development of BEFE. This of course dictated that the language would be object oriented (C++) rather than FORTRAN used in the old BEFE. The idea was to develop a program suitable for problems in geomechanics that did not need to rely on the FEM for dealing with body force effects. Coupling with finite elements was still an issue, but only if it made sense, as for example for the thin shotcrete shell.

The special features required of a simulation program for tunneling are:

- Consideration of inelastic behavior
- Efficient modeling of heterogeneous ground conditions
- Modeling of ground improvement techniques such as grouting, fore poling, pie umbrellas
- Modeling of sequential excavation/construction
- Consideration of supports such as shotcrete, rock bolts, steel arches.

With this in mind the development started more than 10 years ago.

2 THE BEM WITH BODY FORCE EFFECTS

Using the reciprocal theorem by Betti, the following integral equation can be obtained if body forces are present

$$c\mathbf{u} = \int_S \mathbf{U}\cdot\mathbf{t}\cdot dS - \int_S \mathbf{T}\cdot\mathbf{u}\cdot dS + \int_V \mathbf{E}\cdot\boldsymbol{\sigma}_0 \tag{1}$$

Here c is a constant, **u** and **t** are boundary displacements and tractions respectively, **U** and **T** are fundamental solutions for the displacements and tractions, **E** is a tensor of fundamental solutions for the strain and σ_0 is the initial stress tensor, which is considered as the body force here. For the numerical solution of (1) we approximate the boundary with iso-parametric boundary elements and the volume with iso-parametric volume cells. Figure 1 shows the boundary element and cells used for a 3-D analysis. The linear cell (c) will be used for the modeling of rock bolts.

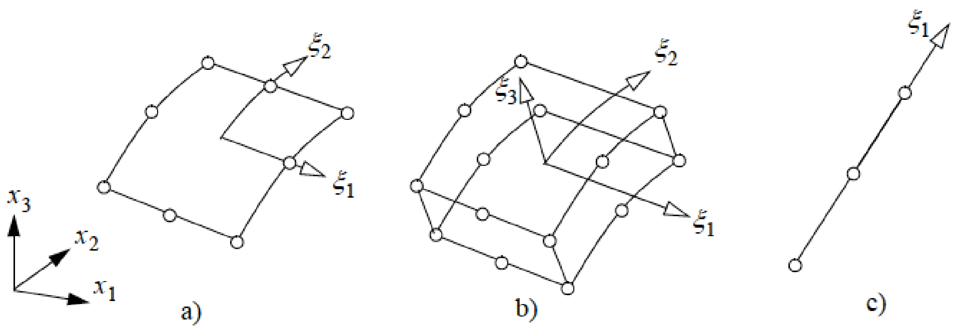

Figure 1 – Boundary element and cells.

Inside a boundary element **u** and **t** are interpolated from the nodal values $\mathbf{u}_i$, $\mathbf{t}_i$ as

$$\mathbf{u} = \sum_{i=1}^{Nodes} N_i\left(\xi_1,\xi_2\right)\mathbf{u}_i \quad , \quad \mathbf{t} = \sum_{i=1}^{Nodes} N_i\left(\xi_1,\xi_2\right)\mathbf{t}_i \tag{2}$$

The initial stresses $\boldsymbol{\sigma}_0$ are interpolated from the nodal values $\boldsymbol{\sigma}_{0i}$ inside a volume cell and inside a linear cell as

$$\boldsymbol{\sigma}_0 = \sum_{i=1}^{Nodes} N_i\left(\xi_1,\xi_2,\xi_3\right)\cdot\boldsymbol{\sigma}_{0i} \quad , \quad \boldsymbol{\sigma}_0 = \sum_{i=1}^{Nodes} N_i\left(\xi_1\right)\cdot\boldsymbol{\sigma}_{0i} \tag{3}$$

Here N_i are suitable linear or quadratic shape functions.

After the discretisation and after applying point collocation (Beer 2008) the following system of equations is obtained

$$\left[\mathbf{T}\right]\left\{\mathbf{u}\right\} = \left[\mathbf{U}\right]\left\{\mathbf{t}\right\} + \left[\mathbf{E}\right]\left\{\boldsymbol{\sigma}_0\right\} \tag{4}$$

where $\left[\mathbf{T}\right]$ and $\left[\mathbf{U}\right]$ are assembled matrices and $\left\{\mathbf{u}\right\}$ and $\left\{\mathbf{t}\right\}$ contain vectors of displacements and tractions at all boundary nodes. The size of the matrices is NxN where N is the number of degrees of freedom on the boundary. $\left[\mathbf{E}\right]$ is a matrix of integrated fundamental solutions for strain and $\left\{\boldsymbol{\sigma}_0\right\}$ is a vector of initial stress components at all cell nodes. The size of the matrix is NxM where M is the total number of stress components at allcell nodes. If either displacements or tractions are known then the system of equations can be solved. If the tunnel excavation is modeled in one step then this is a pure Neumann problem (i.e. **t** is known on the boundary) and the system of equations becomes

$$\left[\mathbf{T}\right]\left\{\mathbf{u}\right\} = \left\{\mathbf{F}\right\} + \left\{\mathbf{F}\right\}_0 \tag{4}$$

where $\left\{\mathbf{F}\right\} = \left[\mathbf{U}\right]\left\{\mathbf{t}\right\}$ and $\left\{\mathbf{F}\right\}_0 = \left[\mathbf{E}\right]\left\{\boldsymbol{\sigma}_0\right\}$.

Once the unknown values at the boundary are determined the displacements at any point inside the continuum may be computed by

$$\mathbf{u} = \left[\mathbf{U}\right]\left\{\mathbf{t}\right\} - \left[\mathbf{T}\right]\left\{\mathbf{u}\right\} + \left[\mathbf{E}\right]\left\{\boldsymbol{\sigma}_0\right\} \tag{5}$$

The strains may be computed from

$$\boldsymbol{\varepsilon} = \left[\tilde{\mathbf{U}}\right]\left\{\mathbf{t}\right\} - \left[\tilde{\mathbf{T}}\right]\left\{\mathbf{u}\right\} + \left[\tilde{\mathbf{E}}\right]\left\{\boldsymbol{\sigma}_0\right\} \tag{6}$$

where $\left[\tilde{\mathbf{U}}\right],\left[\tilde{\mathbf{T}}\right]$ *and* $\left[\tilde{\mathbf{E}}\right]$ are obtained by taking suitable derivatives of (5).

In the following we will concentrate on the determination of the initial stresses and the algorithm for heterogeneous continua.

3 INELASTIC MATERIAL BEHAVIOUR

The treatment of inelastic behavior is similar to the one used for the Finite Element Method. In the BEM either a cell mesh is generated a priori in zones that are likely to exhibit in-elastic behavior or automatically generated as needed.

For the pre-generated cell mesh the procedure is as follows:

1. An elastic analysis is carried out with zero initial stresses by solving $\left[\mathbf{T}\right]\left\{\mathbf{u}\right\} = \left\{\mathbf{F}\right\}$

2. Using (6) and Hooke's law the stresses are computed and the yield condition $F(\sigma)$ is checked at each cell node

3. If $F(\sigma) > 0$ then initial stresses σ_0 are computed (see Beer 2008)

4. $\{\mathbf{F}\}_0$ is assembled from all cell contributions

5. A correction to the displacement field is computed by $[\mathbf{T}]\{\Delta\mathbf{u}\} = \{\mathbf{F}\}_0$

6. The iteration proceeds until the norm of $\{\mathbf{F}\}_0$ is below a given tolerance

For the case of an adaptive generation of a cell mesh a different strategy is applied after step 1:

2. Using stress recovery (Beer 2008) the yield condition $F(\sigma)$ is checked at the nodes of boundary elements.

3. If $F(\sigma) > 0$ then a cell of specified size d is generated using the outward normals.

4. The automatic cell generation continues until all plastic points are covered with cells

The rest is the same as for the case of pre-generated cells. An error analysis together with an automatic cell refinement can be implemented to assure that the cell mesh does not introduce any additional error (see Thoeni, 2009).

4 HETEROGENOUS GROUND CONDITIONS, INCLUSIONS

With the classical BEM only homogeneous ground conditions can be considered. Piecewise heterogeneous conditions may be considered with the multi-region BEM. However, the effort in mesh generation and the solution increases considerably with the heterogeneity, because the interfaces between different materials have to be specified and the number of unknowns is increased. For practical applications in tunnelling therefore an alternative method is required.

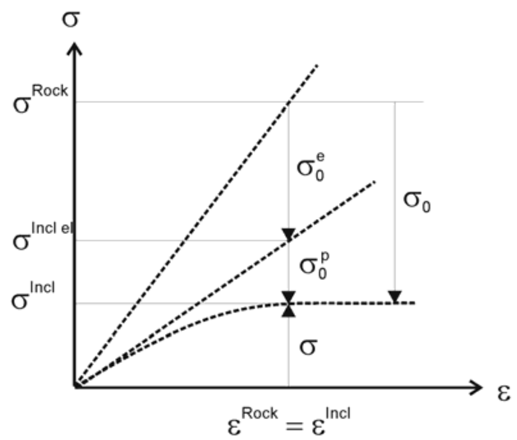

Figure 2 – Computation of initial stress for inclusion.

The proposed method basically uses a similar approach as for plasticity and the problem is solved iteratively. The heterogeneities are treated as inclusions and discretised with cells. The algorithm is as follows:

1. An elastic analysis is carried out with zero initial stresses by solving $[\mathbf{T}]\{\mathbf{u}\}=\{\mathbf{F}\}$

2. Using (6) the strain tensor ε is computed at cell nodes. If the elastic properties of the continuum (here specified as Rock) and the inclusion (Incl) are different then the strain will still be the same but the stress will be different. This difference in stress is computed as shown in Fig. 2 and applied as initial stress σ_0^e. Additionally, if $F(\sigma)>0$ at this point then an additional initial stress σ_0^p is computed.

3. $\{\mathbf{F}\}_0$ is assembled using the total initial stress at all cell nodes

   The iteration proceeds exactly as for the inelastic material.

This method can be applied to geological conditions and ground improvement techniques such as grouting and fore-poling. It may also be used for modeling rock bolts. Because rock bolts have a small diameter to length ratio the volume integral can be split up into an integral over the cross-section of the bolt which is evaluated analytically and an integral over the length of the bolt which is evaluated numerically using the linear cells in Fig. 1 (Riederer, 2010). It is possible to consider pre-stressing and either fully grouted or partially grouted rock bolts (Riederer, 2009).

5 SEQUENTIAL EXCAVATION

For tunneling in difficult ground tunnel construction has to be carried out in stages with excavation of part of the tunnel followed by the installation of supports. Sequential excavation can be modeled by the multi-region method but the effort and the number of unknowns increases significantly with the complexity of the excavation sequence. An alternative method requiring significantly less effort and a considerably smaller number of unknowns is presented here.

The excavation process in a pre-stressed ground means that material is removed that has previously supplied support. Because of the removal of this support a lack of equilibrium exists that is restored by a change in stress. In the simulation the removal of material is modeled as follows (see Fig. 3):

1. The stress $\mathbf{t}_n$ on the boundary that will be exposed in the following step n+1 is computed.

2. The boundary element mesh is extended to represent the excavation boundary for step n+1 and the boundary stress computed in 1) is applied in the opposite direction.

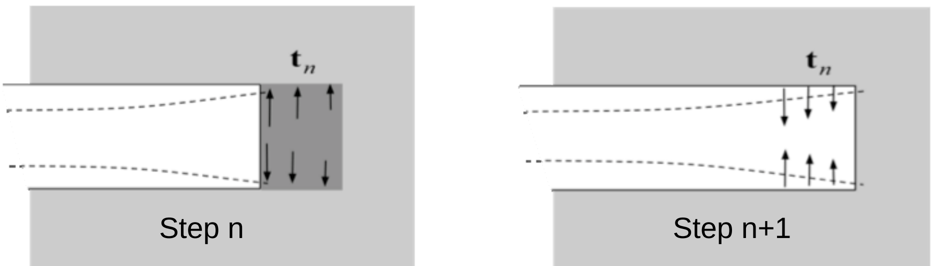

Figure 3 – Simulation of excavation.

This method for the simulation of excavation is very efficient as a large number of interface unknowns is avoided and one starts with a small mesh that grows as the tunnel advances.

6 MODELLING OF SUPPORT

In tunnelling the main types of support used are shotcrete, rock bolts and steel arches. The modelling of rock bolts was already discussed in Chapter 4 together with inclusions. For the thin shotcrete shell and the steel arches the use of shell and curved beam finite elements seems to the most efficient approach. The curved beam and shell finite elements are depicted in Fig. 4. They possess 3 translational and 3 rotational degrees of freedom at each node.

The coupling of finite and boundary elements is carried out in the following way:

1. A stiffness of the boundary element region is computed solving a Dirichlet problem (unit value of displacement at a node, all other displacements zero) N times where N is the number of degrees of freedom of the interface. For each problem N tractions are computed resulting in an NxN matrix of interface tractions
2. The tractions are converted into work equivalent nodal point forces, resulting in a stiffness matrix of the interface (Beer, 2008)
3. The translational degrees of freedom of the BEM region are coupled with the corresponding translational degrees of freedom of the shell or beam finite element. The rotational degrees of freedom are only coupled between the shell/beam element.

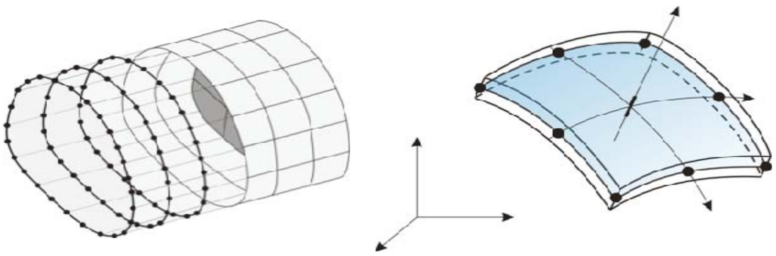

Figure 4 – Curved beam and thin shell finite elements.

7 TEST EXAMPLES

Here several test examples are presented that show that the accuracy that can be obtained with the proposed method is good and equal or better than the one obtained using established methods such as finite element or finite difference methods.

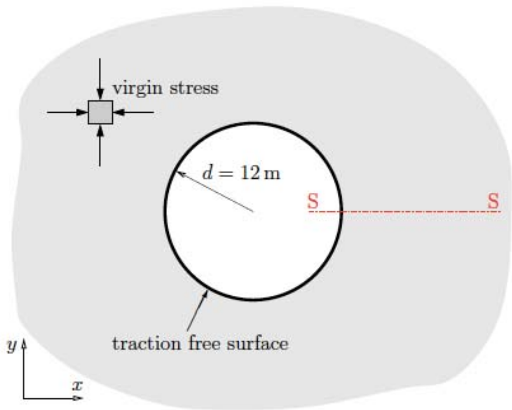

Figure 5 – Test example for plasticity.

7.1 Non-linear material behaviour

Here we show an example of the non-linear solution of a circular excavation using the adaptive cell refinement algorithm. The example in Fig. 5 is a circular excavation in a pre-stressed medium. The material law for the ground is the Drucker-Prager yield condition.

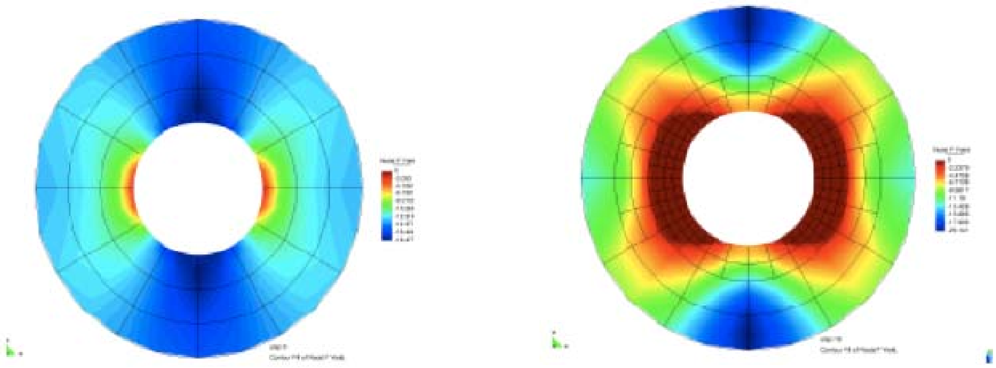

Figure 6 – Initial and adaptive cell mesh with contours of yield function value.

In Fig. 6 the initial and the adaptive cell mesh is shown and in Fig. 7 a comparison of the vertical stress along the line S-S is made. The reference solutions are with very fine FEM or BEM meshes. It can be seen that even with a coarse cell mesh good results can be obtained.

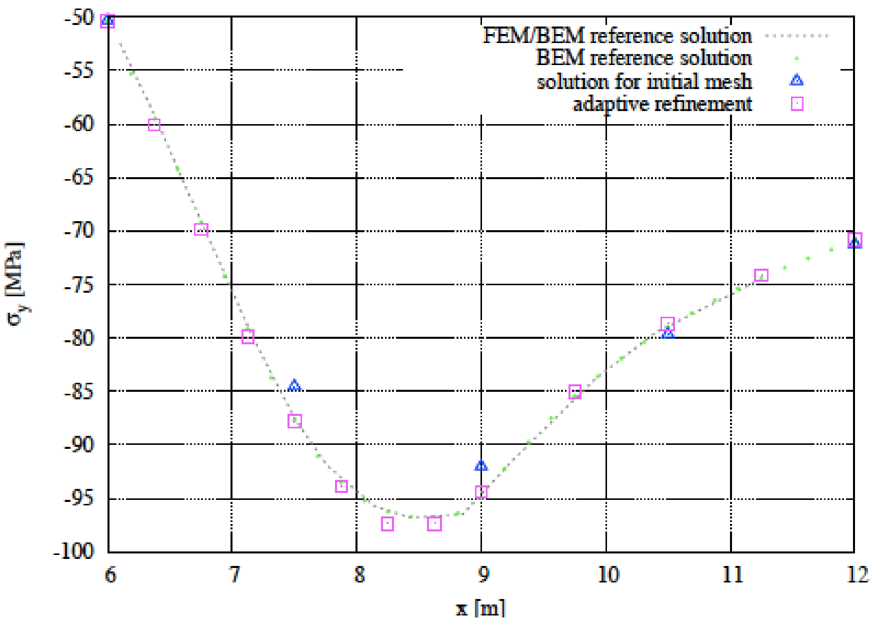

Figure 7 – Comparison of vertical stress.

7.2 Heterogeneous ground and rock bolts

Here we present an example of a tunnel in heterogeneous ground conditions. The ground is assumed to contain a thin fault zone of different material. In addition 3 anchors are modelled that transsect this zone. A finite element analysis with the program PLAXIS was carried out where the whole domain has to be discretised and the anchors are modelled by truss elements. In the BEM only the fault zone and the anchors are modelled by cells.

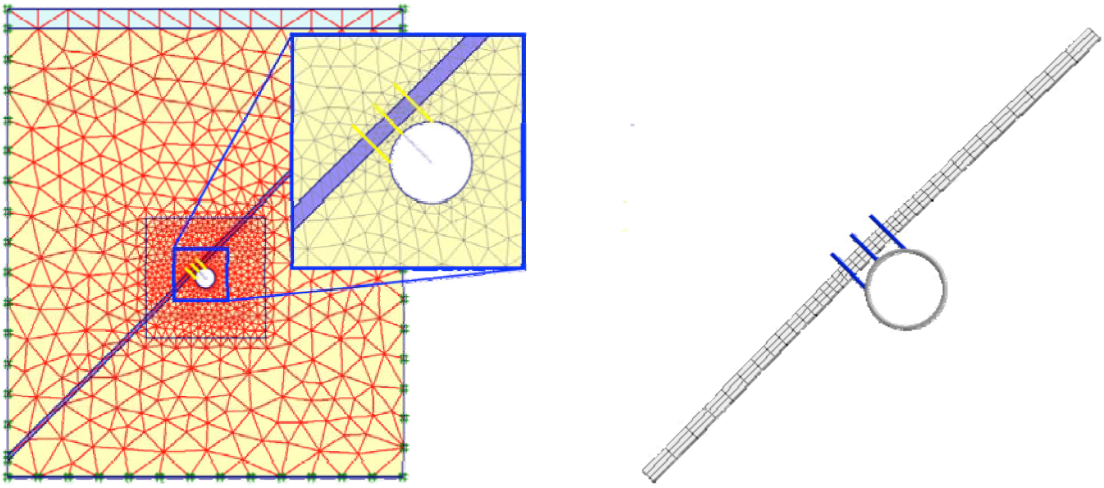

Figure 8 – Test example for heterogeneous ground conditions: Left finite element mesh, right BEM mesh.

Fig. 9 shows a comparison of the results and it can be seen that good agreement can be obtained

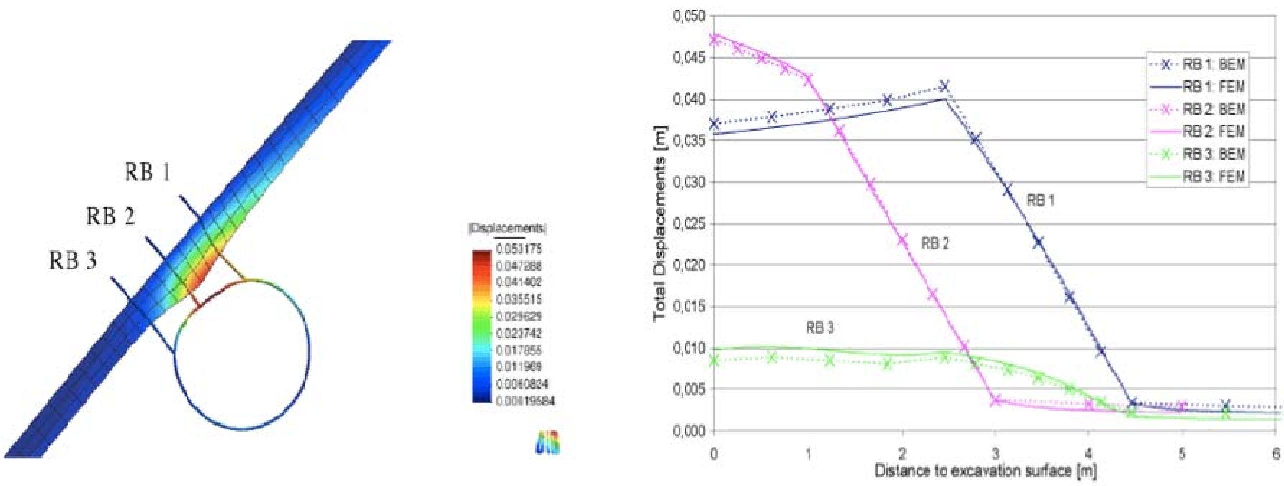

Figure 9 – Displaced shape (exaggerated) and comparison of displacements between FEM and BEM.

7.3 Modeling of shotcrete

The modelling of shotcrete is rather complicated because shotcrete has a very low strength when it is placed and only attains strength after a certain time. For the simulation in conventional (drill&blast) tunnel construction it is very important to consider this. The setting time of shotcrete is influenced by many factors such as the chemical composition of cement and additives etc. An important aspect is if water is allowed to drain freely (drained conditions) or not (sealed conditions). The test example presented here is the analysis of a circular excavation, which is sequentially constructed, i.e. in a sequence of excavation and shotcreting steps.
The mesh used for the analysis is shown in Fig. 8. The excavation was made in a virgin vertical compressive stress of 2.75 MPa with a multiplication factor of 0.5 for the horizontal stress components. The results are shown in Fig. 9 where a comparison between the case without shotcrete and with shotcrete support with sealed conditions and with drained conditions is made. One can see the supporting potential of shotcrete and the differnec between sealed and drained conditions clearly. In addition the presence of steel ribs has been considered and their support potential shown.

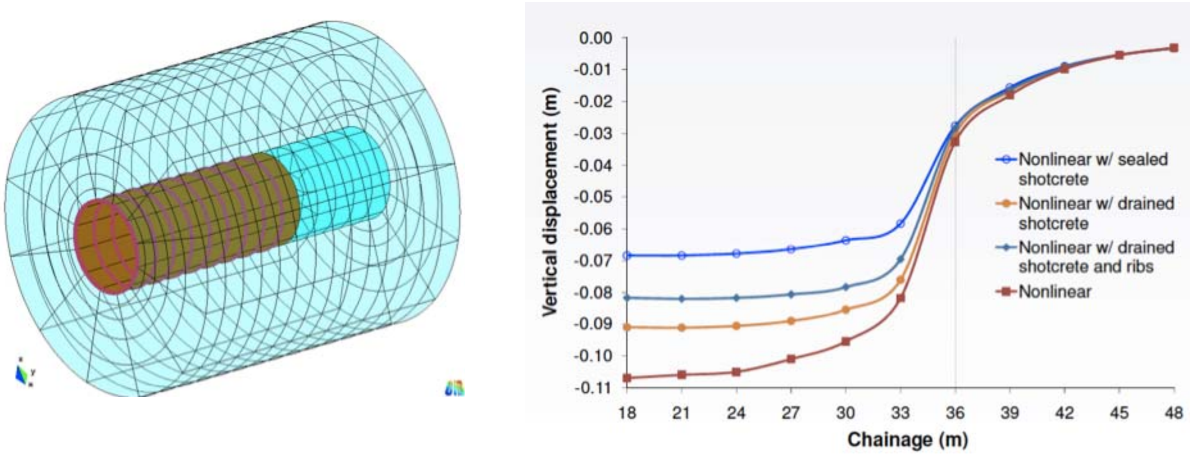

Figure 10 – Shotcrete test example: Mesh and results.

7.4 Sequential excavation

The final test example is designed to show that the proposed method for the sequential excavation gives good results. It relates to the top heading/bench excavation of a real tunnel. The mesh and the excavation sequence are indicated in Fig. 11.

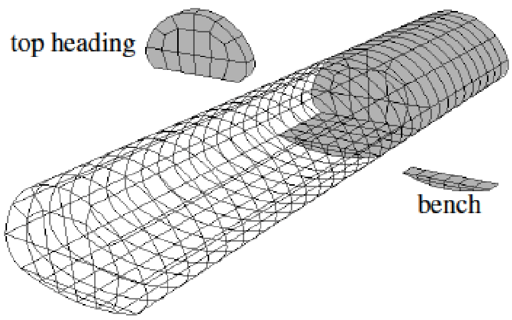

Figure 11 – Sequential excavation test example: Mesh and results.

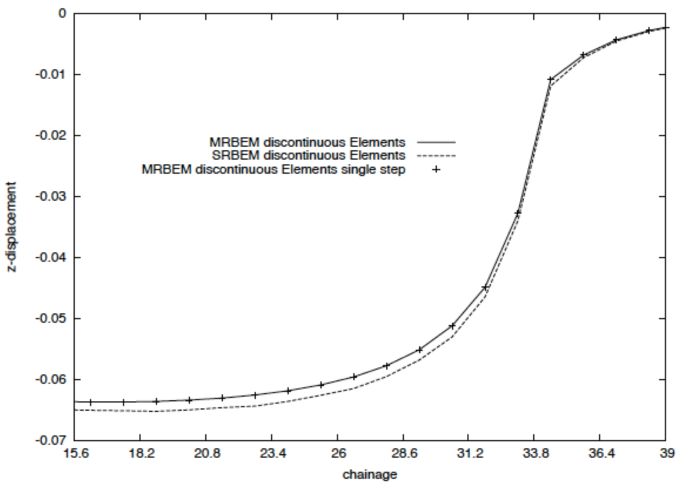

Figure 12 – Convergence curves for the different methods.

The excavation was simulated first by the multi-region method (MRBEM), then by the proposed single-region method (SRBEM) and finally with a single step. The ground was assumed to behave elastically therefore all methods should give the same results at the end, when the tunnel is fully excavated. It can be seen that the proposed method gives good results.

8 PRACTICAL APPLICATION

The practical application relates to the construction of the Koralm tunnel in Austria.

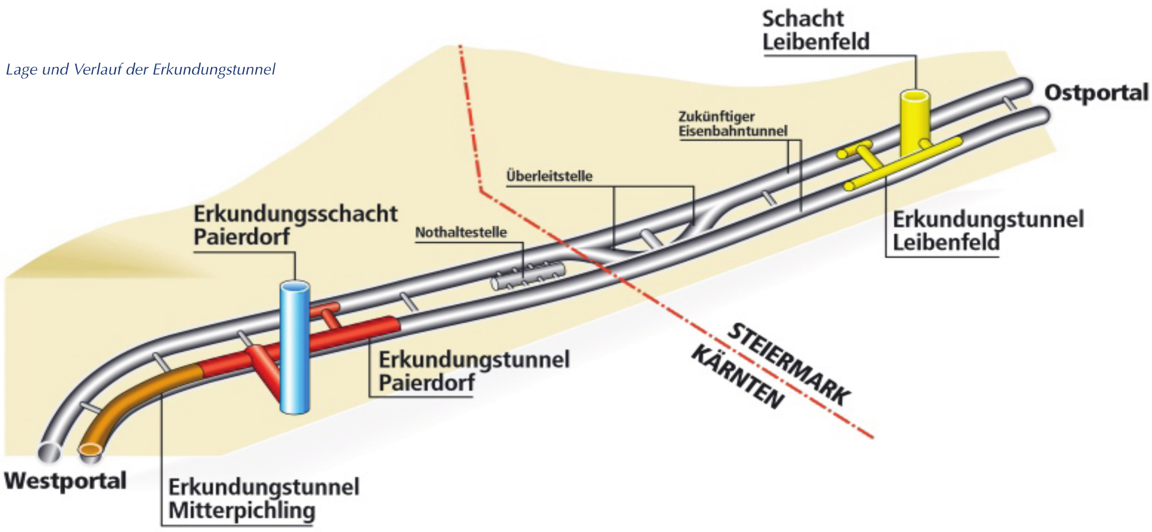

Figure 13 – Layout of the Koralm tunnel.

For the simulation, part of the pilot tunnel highlighted in red was chosen. Here the excavation was carried out in 2 stages (top heading, bench).

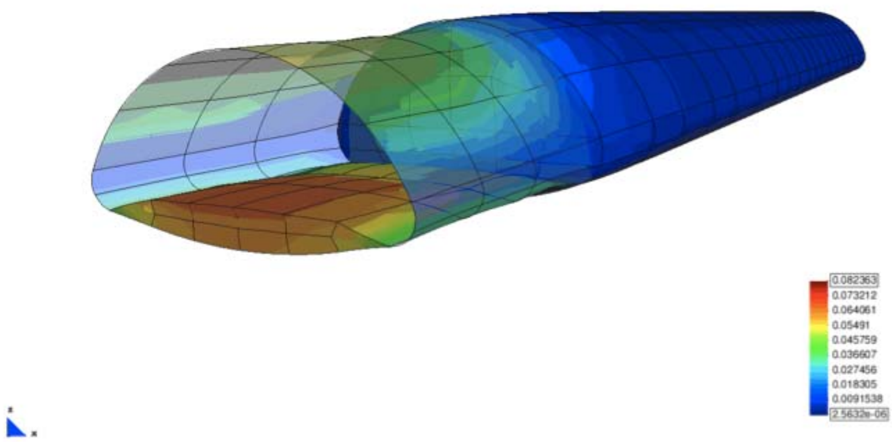

Figure 14 – Displaced shape and contours of displacement.

Fig. 14 shows the displaced shape and contours of displacement. In Fig. 15 a comparison is made between measured and observed displacements. It can be seen that good agreement is obtained.

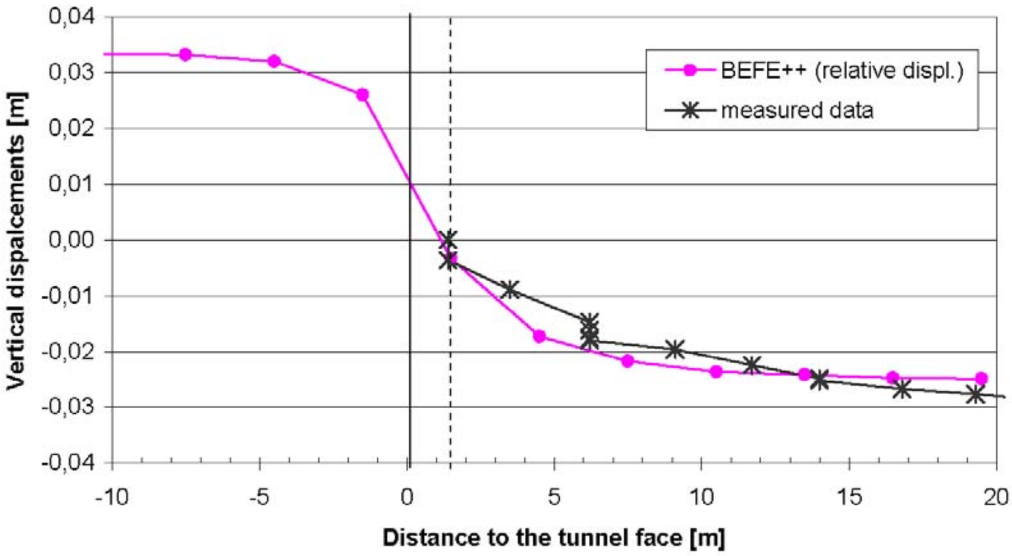

Figure 15 – Comparison of simulated and measured crown displacements.

9 CONCLUSIONS

An overview of the development of a special program BEFE++ for the simulation of tunnel excavation was presented. To make the Boundary Element Method fit for the simulation in geomechanics several theoretical extensions of the method were necessary. Based on the original ideas of Venturini, efficient methods were developed for the simulation of heterogeneous material, sequential construction and for modelling the various types of ground support used in tunnelling. Test examples demonstrate that the proposed methods work well and give as good if not better results than obtained from currently used simulation programs. Finally a practical application was shown. Further details may be obtained from the cited literature. It is hoped that the paper will stimulate the use of the BEM in geomechanics and that the spirit of innovation in this field, originated by Sergio Venturini will continue in the future.

10 ACKNOWLEDGEMENTS

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